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# Method for measuring two-qubit entanglement of formation by local operations and classical communication

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**Abstract**

We present a parametrically efficient method for measuring the entanglement of formation $E_{\mathrm{f}}$ in an arbitrarily given unknown two-qubit state $\rho_{\mathrm{AB}}$ by local operations and classical communication. The two observers, Alice and Bob, first perform some local operations on their composite systems separately, by which the desired global quantum states can be prepared. Then they estimate seven functions via two modified local quantum networks supplemented a classical communication. After obtaining these functions, Alice and Bob can determine the concurrence $C$ and the entanglement of formation $E_{\mathrm{f}}$.

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## 1. Introduction

In the development of quantum mechanics, entanglement plays a significant role [1–3]. Nowadays, it has been rediscovered as a new physical resource for quantum information processing [4]. Before using entanglement in a given *unknown* system, one needs to make sure that it really exists. Furthermore, one wants to know how much entanglement is there. The simplest way to approach this, without new ideas, is prior state reconstruction by quantum tomography [5] which provides full knowledge about the density matrix of the system. However, there are more efficient ways, by which one can compute the entanglement properties directly. Sancho and Huelga [6] presented the first direct method for determining the concurrence [7] of two-qubit pure state. Ekert, Horodecki and co-workers did a series of works on entanglement detection and measurement in an unknown mixed state without the prior state reconstruction [8–10].

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In particular, it is important in the implementation of quantum communication to detect and measure entanglement within local operations and classical communication (LOCC) scenario, in which the two observers, Alice and bob, are far apart from each other and share the unknown composite system. It has been proven that entanglement is a precondition for secure quantum key distribution [11]. Recently, Alves *et al* [12] did two works: the first is that Alice and Bob can estimate the functions of composite quantum states by two local networks[3]; the second is that they prove the physical operation $\widetilde{I_{\rm A} \otimes \Lambda_{\rm B}}$ [9] can be implemented by LOCC where $\Lambda_{\rm B}$ is a nonphysical map acting on the subsystem at Bob's location. The authors applied the techniques to the detection of entanglement and estimation of channel capacity. However, in order to make use of entanglement better, the well-defined entanglement of formation in the composite system need to be considered.

In [10], using the structural physical approximation (SPA) [13] of partial transposition map, Horodecki presented an efficient and experimentally viable method for measuring two-qubit entanglement of formation without the prior state reconstruction. In this paper, we present an LOCC method, an extension of Horodecki's method. The paper is organized as follows: in section 2, we characterize the LOCC method in detail. We also analyse the efficiency of our method against the LOCC quantum tomography. Finally, in section 3, we conclude the paper with a summary.

## 2. Method for measuring two-qubit entanglement of formation by LOCC

For a two-qubit state $\rho_{\rm AB}$, the formula for the entanglement of formation [7] is

$$E_f(\rho_{\rm AB}) = h\left(\frac{1+\sqrt{1-C(\rho_{\rm AB})^2}}{2}\right), \qquad (1)$$

where $h(x)$ is Shannon function, and $C$ denotes the concurrence defined as

$$C(\rho_{\rm AB}) = \max[\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}, 0], \qquad (2)$$

in which the four monotonically decreasing real numbers $\{\lambda_i\}$ are the eigenvalues of the matrix $\rho_{\rm AB}\widetilde{\rho_{\rm AB}}$, here $\widetilde{\rho_{\rm AB}} = (\sigma_y \otimes \sigma_y)\rho^*_{\rm AB}(\sigma_y \otimes \sigma_y)$. Both $C$ and $E_{\rm f}$ for an arbitrarily given two-qubit state $\rho_{\rm AB}$ can be determined as long as one knows these eigenvalues. P. Horodecki presented a global method [10] for measuring the entanglement of formation $E_{\rm f}$. On the basis of the relation $\mathrm{Tr}[(\rho_{\rm AB}\widetilde{\rho_{\rm AB}})^k] = \sum_i(\lambda_i)^k$, he pointed out that one can get $\{\lambda_i\}$ by estimating four parameters $\mathrm{Tr}[(\rho_{\rm AB}\widetilde{\rho_{\rm AB}})^k]$, $k = 1, 2, 3, 4$.

In our LOCC method, Alice and Bob can get the four parameters by measuring seven functions. It is assumed that they share a number of the unknown two-qubit quantum states $\rho_{\rm AB}$. First, Alice and Bob divide their initial systems into four groups and prepare the requisite input states. Then they estimate seven functions of the input states by two local collective measurements. After obtaining all of the seven functions, they can determine the entanglement of formation $E_{\rm f}$. The detailed description of our LOCC method is presented in the following two subsections.

### *2.1. Preparation of the requisite global quantum states*

Alice and Bob divide their initial ensemble into four groups. In the $k$th group, they subdivide the quantum states into sets of $2k$ elements, for $k = 1, 2, 3, 4$. For example, in the second group, they subdivide the quantum states into sets of $(\rho_{\rm AB} \otimes \rho_{\rm AB})^{\otimes 2}$.

[3] In section 2.2, we will see that the two local networks are only valid when the input quantum states have some symmetrical property.

In the LOCC method, Alice and Bob need the input states $(\rho_{\mathrm{AB}}\otimes\widetilde{\rho_{\mathrm{AB}}})^{\otimes k}$ for $k=1,2,3,4$. However, the transformation from $\rho_{\mathrm{AB}}$ to $\widetilde{\rho_{\mathrm{AB}}}$ is nonphysical. To see how to prepare these quantum states, we need to recall the concept of SPA [13].

Horodecki pointed out that if one mixes a nonphysical map $\Lambda$ with an appropriate proportion depolarizing map $D$, then the resulting map [13],

$$\widetilde{\Lambda} = pD + (1-p)\Lambda, \tag{3}$$

will be completely positive, and, as such, it represents a physically allowed transformation. The effect of the depolarizing map $D$ on a dimension-$d$ quantum state $\rho$ is

$$D(\rho) = I/d, \tag{4}$$

i.e., it turns any $\rho$ into a maximal mixed state [13]. Recently, Alves *et al* [12] have proven that the SPA $\widetilde{I_{\mathrm{A}}\otimes\Lambda_{\mathrm{B}}}$ can be implemented by LOCC if it can be written as a convex sum

$$\widetilde{I_{\mathrm{A}}\otimes\Lambda_{\mathrm{B}}} = \sum_i p_i M_{\mathrm{A}_i}\otimes N_{\mathrm{B}_i}, \tag{5}$$

where the physical operation $M_{\mathrm{A}_i}$ acts on the subsystems of Alice and the physical operation $N_{\mathrm{B}_i}$ acts on the subsystems of Bob.

In the procedure of quantum states preparation, Alice and Bob need to implement the SPA of nonphysical map $(I_{\mathrm{AB}}\otimes T_{\mathrm{AB}})^{\otimes k}$ on the quantum states $(\rho_{\mathrm{AB}}\otimes\rho_{\mathrm{AB}})^{\otimes k}$. But, after some analysis, we find that the SPA cannot be written as the form of equation (5). Fortunately, Alice and Bob can implement it indirectly. For convenience, let the notation $\mathrm{S}(\Gamma)$ denotes the SPA of a nonphysical map $\Gamma$. First of all, we briefly explain that the operation $\mathrm{S}\big(I_{\mathrm{A}_{1k}}\otimes I_{\mathrm{B}_{1k}}\otimes I_{\mathrm{A}_{2k}}\otimes T_{\mathrm{B}_{2k}}\big)$ (the operators with suffixes $1k$ and $2k$ represent the maps on the quantum states in the odd and even position, respectively) can be implemented by LOCC,

$$\begin{aligned}\mathrm{S}\big(I_{\mathrm{A}_{1k}}\otimes I_{\mathrm{B}_{1k}}\otimes I_{\mathrm{A}_{2k}}\otimes T_{\mathrm{B}_{2k}}\big) &= \alpha D_{\mathrm{A}_{1k}}\otimes D_{\mathrm{B}_{1k}}\otimes D_{\mathrm{A}_{2k}}\otimes D_{\mathrm{B}_{2k}} + (1-\alpha) I_{\mathrm{A}_{1k}}\otimes I_{\mathrm{B}_{1k}}\otimes I_{\mathrm{A}_{2k}}\otimes T_{\mathrm{B}_{2k}}\\ &= (\alpha-\beta)\Theta_{\mathrm{A}_{1k}\mathrm{A}_{2k}}\otimes D_{\mathrm{B}_{1k}\mathrm{B}_{2k}} + (1-\alpha+\beta) I_{\mathrm{A}_{1k}\mathrm{A}_{2k}}\otimes \Xi_{\mathrm{B}_{1k}\mathrm{B}_{2k}},\end{aligned}$$

where

$$\begin{aligned}\Theta_{\mathrm{A}_{1k}\mathrm{A}_{2k}} &= \frac{\alpha}{\alpha-\beta} D_{\mathrm{A}_{1k}}\otimes D_{\mathrm{A}_{2k}} + \frac{-\beta}{\alpha-\beta} I_{\mathrm{A}_{1k}}\otimes I_{\mathrm{A}_{2k}},\\ \Xi_{\mathrm{B}_{1k}\mathrm{B}_{2k}} &= \frac{1-\alpha}{1-\alpha+\beta} I_{\mathrm{B}_{1k}}\otimes T_{\mathrm{B}_{2k}} + \frac{\beta}{1-\alpha+\beta} D_{\mathrm{B}_{1k}}\otimes D_{\mathrm{B}_{2k}}.\end{aligned} \tag{6}$$

The operators $\Theta_{\mathrm{A}_{1k}\mathrm{A}_{2k}}$ and $\Xi_{\mathrm{B}_{1k}\mathrm{B}_{2k}}$ will be completely positive when $\alpha \geqslant d_k^7/\big(d_k^7+1\big)$ and $\beta \geqslant d_k^3/\big(d_k^7+1\big)$ (for the optimal implementation we should choose equal mark), in which $d_k$ is the dimension of subsystem. In fact, equation (6) is a special case in the discussion of Alves *et al* [12], in which we choose $\Lambda_{\mathrm{B}} = I_{\mathrm{B}_{1k}}\otimes T_{\mathrm{B}_{2k}}$. In a similar way, we can validate that the operation $\mathrm{S}\big(I_{\mathrm{A}_{1k}}\otimes I_{\mathrm{B}_{1k}}\otimes T_{\mathrm{A}_{2k}}\otimes I_{\mathrm{B}_{2k}}\big)$ can also be implemented by LOCC when we choose the same parameters.

Now we describe the procedure of quantum state preparation. For example, Alice and Bob have chosen a set of quantum state in the $k$th group. The procedure consists of three kinds of physical operations. First, they perform the optimal operation $\Omega_k' = \mathrm{S}\big(I_{\mathrm{A}_{1k}}\otimes I_{\mathrm{B}_{1k}}\otimes I_{\mathrm{A}_{2k}}\otimes T_{\mathrm{B}_{2k}}\big)$ on the set of $(\rho_{\mathrm{AB}}\otimes\rho_{\mathrm{AB}})^{\otimes k}$. After doing this operation, they can get the following quantum state

$$\begin{aligned}\rho_k' &= \Omega_k'[(\rho_{\mathrm{AB}}\otimes\rho_{\mathrm{AB}})^{\otimes k}]\\ &= \frac{d_k^3}{d_k^7+1} I_{\mathrm{A}_{1k}\mathrm{B}_{1k}}\otimes I_{\mathrm{A}_{2k}\mathrm{B}_{2k}} + \frac{1}{d_k^7+1}\big(\rho_{\mathrm{AB}}\otimes\rho_{\mathrm{AB}}^{T_{\mathrm{B}}}\big)^{\otimes k},\end{aligned} \tag{7}$$

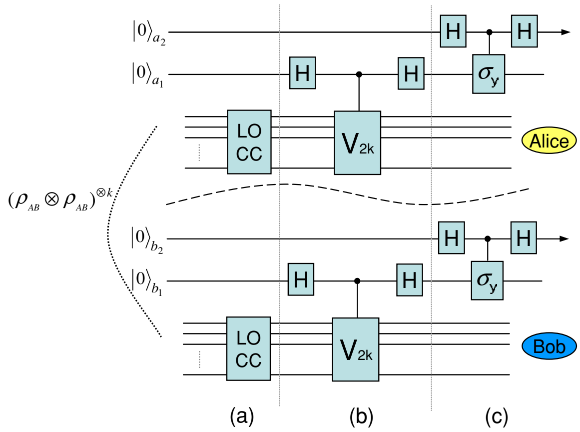


**Figure 1.** Networks for remote measurement of the parameters $\mathrm{Tr}[(\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}})^k]$. Part (a) denotes the preparation procedure of $\rho_k$, part (b) is the network presented in [12] and part (c) is the additional part.

where $d_k = 2^k$. Second, they perform the optimal operation $\Omega''_k = \mathrm{S}\big(I_{\mathrm{A}_{1k}} \otimes I_{\mathrm{B}_{1k}} \otimes T_{\mathrm{A}_{2k}} \otimes I_{\mathrm{B}_{2k}}\big)$ on the quantum states $\rho'_k$. The output states will be

$$\begin{aligned}\rho''_k &= \Omega''_k(\rho_k)\\ &= \delta I_{\mathrm{A}_{1k}B_{1k}} \otimes I_{\mathrm{A}_{2k}\mathrm{B}_{2k}} + \gamma(\rho_{\mathrm{AB}} \otimes \rho^*_{\mathrm{AB}})^{\otimes k},\end{aligned} \tag{8}$$

where $\delta = d_k^3\big(d_k^7+2\big)\big/\big(d_k^7+1\big)^2$ and $\gamma = 1\big/\big(d_k^7+1\big)^2$. In equation (8), we have used the property: $\rho_{\mathrm{AB}}^{T_{\mathrm{A}}T_{\mathrm{B}}} = \rho_{\mathrm{AB}}^T = \rho^*_{\mathrm{AB}}$. Finally, Alice and Bob perform the 'spin-flip' operation $\sigma_y$ respectively on each quantum state which is in the even position. As a result, they will get the quantum state

$$\rho_k = \delta I_{\mathrm{A}_{1k}\mathrm{B}_{1k}} \otimes I_{\mathrm{A}_{2k}\mathrm{B}_{2k}} + \gamma\,(\rho_{\mathrm{AB}} \otimes \widetilde{\rho_{\mathrm{AB}}})^{\otimes k}. \tag{9}$$

After performing the above operations, Alice and Bob can get the quantum state $\rho_k$. Although $\rho_k$ is not equal to the state $(\rho_{\mathrm{AB}} \otimes \widetilde{\rho_{\mathrm{AB}}})^{\otimes k}$, the two states have the same direction in the generalized Bloch representation, which is the essence of the SPA [13]. When Alice and Bob choose the set in different groups, they can get the desired input states $\rho_k$, for $k = 1, 2, 3, 4$.

### 2.2. *Estimation of the parameters* $\mathrm{Tr}[(\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}})^k]$

In [12], Alves *et al* presented two local networks which can estimate the functions of non-local composite quantum state. However, the networks are not universal, they require the quantum state to satisfy some symmetrical property. Here we present two modified networks which can estimate the functions of an arbitrarily given quantum state. As shown in figure 1, part (a) denotes the preparation procedure of input state $\rho_k$, part (b) is the network presented in [12] and part (c) is the additional part. In our LOCC method, Alice and Bob can estimate the four parameters $\mathrm{Tr}[(\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}})^k]$ for $k = 1, 2, 3, 4$, via the modified networks, which needs to measure seven functions of the input quantum states.

Now we reanalyse part (b); in this part, the input state is

$$\rho_{\mathrm{in}}(k) = \rho_k \otimes \rho_{a_1} \otimes \rho_{b_1}, \tag{10}$$

where $\rho_{a_1} = |0\rangle\langle 0|_{a_1}$ and $\rho_{b_1} = |0\rangle\langle 0|_{b_1}$ are the initial states of the ancillary qubits in Alice and Bob's networks, respectively. In the computational basis, the Hadamard gate and the controlled-$V_{2k}$ gate within Alice and Bob's networks are denoted by

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}, \qquad U_{C-V_{2k}} = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\otimes I_{2k} + \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\otimes V_{2k}, \quad (11)$$

where $V_{2k}$ is the shift operator and has the property $\mathrm{Tr}(V_{2k}\rho_1\otimes\rho_2\otimes\ldots\otimes\rho_{2k}) = \mathrm{Tr}(\rho_1\rho_2\ldots\rho_{2k})$ [10]. Passing through part (b), the incoming state transforms into the following state,

$$\rho'_{\mathrm{out}}(k) = U_{h_1}U_v U_{h_1}\rho_{\mathrm{in}}(k)U_{h_1}^\dagger U_v^\dagger U_{h_1}^\dagger, \quad (12)$$

where $U_{h_1} = H_{a_1}\otimes H_{b_1}\otimes(I_{\mathrm{AB}}\otimes I_{\mathrm{AB}})^{\otimes k}$ and $U_v = U_{C-V_{A2k}}\otimes U_{C-V_{B2k}}$. In the state $\rho'_{\mathrm{out}}(k)$, what we care about is the quantum state evolution of the two ancillary qubits $a_1$ and $b_1$. After some deduction, we can get

$$\rho_{a_1b_1}(k) = \mathrm{Tr}_{\mathrm{AB}}[\rho'_{\mathrm{out}}(k)]$$
$$= \frac{1}{4}\begin{pmatrix} 1+\mu_1^{(k)}+\mu_3^{(k)} & \mu_5^{(k)} & -\mu_5^{(k)} & \mu_4^{(k)}\\ -\mu_5^{(k)} & 1+\mu_2^{(k)}-\mu_3^{(k)} & -\mu_4^{(k)} & \mu_5^{(k)}\\ \mu_5^{(k)} & -\mu_4^{(k)} & 1-\mu_2^{(k)}-\mu_3^{(k)} & -\mu_5^{(k)}\\ \mu_4^{(k)} & -\mu_5^{(k)} & \mu_5^{(k)} & 1-\mu_1^{(k)}+\mu_3^{(k)} \end{pmatrix}$$

where

$$\begin{aligned}
\mu_1^{(k)} &= \mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}\otimes I_{\mathrm{B}_{2k}}\big)\rho_k\big] + \mathrm{Tr}\big[\big(I_{\mathrm{A}_{2k}}\otimes V_{\mathrm{B}_{2k}}\big)\rho_k\big],\\
\mu_2^{(k)} &= \mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}\otimes I_{\mathrm{B}_{2k}}\big)\rho_k\big] - \mathrm{Tr}\big[\big(I_{\mathrm{A}_{2k}}\otimes V_{\mathrm{B}_{2k}}\big)\rho_k\big],\\
\mu_3^{(k)} &= \tfrac{1}{2}\,\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}\otimes V_{\mathrm{B}_{2k}}\big)\rho_k\big] + \tfrac{1}{4}\,\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}^\dagger\otimes V_{\mathrm{B}_{2k}}\big)\rho_k\big] + \tfrac{1}{4}\,\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}\otimes V_{\mathrm{B}_{2k}}^\dagger\big)\rho_k\big],\\
\mu_4^{(k)} &= \tfrac{1}{2}\,\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}\otimes V_{\mathrm{B}_{2k}}\big)\rho_k\big] - \tfrac{1}{4}\,\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}^\dagger\otimes V_{\mathrm{B}_{2k}}\big)\rho_k\big] - \tfrac{1}{4}\,\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}\otimes V_{\mathrm{B}_{2k}}^\dagger\big)\rho_k\big],\\
\mu_5^{(k)} &= \tfrac{1}{4}\,\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}^\dagger\otimes V_{\mathrm{B}_{2k}}\big)\rho_k\big] - \tfrac{1}{4}\,\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}\otimes V_{\mathrm{B}_{2k}}^\dagger\big)\rho_k\big].
\end{aligned} \quad (13)$$

In equation (13), we have used $\mathrm{Tr}[U^\dagger\rho] = (\mathrm{Tr}[U\rho])^*$ and the functions $\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}\otimes I_{\mathrm{B}_{2k}}\big)\rho_k\big]$, $\mathrm{Tr}\big[\big(I_{\mathrm{A}_{2k}}\otimes V_{\mathrm{B}_{2k}}\big)\rho_k\big]$, $\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}\otimes V_{\mathrm{B}_{2k}}\big)\rho_k\big]$ are real. If Alice and Bob measure the expectation value of $\sigma_z\otimes\sigma_z$ on the state $\rho_{a_1b_1}(k)$, they will get $\mu_3^{(k)} = \mathrm{Tr}[(\sigma_z\otimes\sigma_z)\rho_{a_1b_1}(k)]$ (the measurement needs a classical communication).

When the two observers choose different input state $\rho_{\mathrm{in}}(k)$ for $k = 1, 2, 3, 4$, they can obtain the functions $\mu_3^{(1)}, \mu_3^{(2)}, \mu_3^{(3)}, \mu_3^{(4)}$. It should be noted that the shift operator $V_{2k}$ *is not* Hermitian for $k \geqslant 2$ [13]. Combining equations (9) and (13), we can obtain

$$\begin{aligned}
\mu_3^{(1)} &= 4\delta + \gamma\,\mathrm{Tr}[\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}}],\\
\mu_3^{(k)} &= 4\delta + \frac{\gamma}{2}\,\mathrm{Tr}[(\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}})^k] + \frac{\gamma}{4}\big(\mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}^\dagger\otimes V_{\mathrm{B}_{2k}}\big)(\rho_{\mathrm{AB}}\otimes\widetilde{\rho_{\mathrm{AB}}})^{\otimes k}\big] + \mathrm{c.c.}\big), \qquad k = 2, 3, 4.
\end{aligned} \quad (14)$$

According to equation (14), Alice and Bob can obtain the parameter $\mathrm{Tr}[\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}}]$ in terms of $\mu_3^{(1)}$. However, they *cannot* get the parameters $\mathrm{Tr}[(\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}})^k]$ for $k = 2, 3, 4$, except that the quantum state $\rho_{\mathrm{AB}}$ has some symmetric property which makes the following equation hold,

$$\mathrm{Tr}[(\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}})^k] = \mathrm{Tr}\big[\big(V_{\mathrm{A}_{2k}}^\dagger\otimes V_{\mathrm{B}_{2k}}\big)(\rho_{\mathrm{AB}}\otimes\widetilde{\rho_{\mathrm{AB}}})^{\otimes k}\big], \qquad k = 2, 3, 4. \quad (15)$$

In order to get the parameters $\mathrm{Tr}[(\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}})^2]$, $\mathrm{Tr}[(\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}})^3]$ and $\mathrm{Tr}[(\rho_{\mathrm{AB}}\widetilde{\rho_{\mathrm{AB}}})^4]$ for an arbitrarily given quantum state, Alice and Bob need make other measurements with the modified networks. As shown in figure 1, they add part (c) after part (b). In part (c), the input state is $\rho_{\rm in}^{c}(k) = \rho_{a_1b_1}(k) \otimes \rho_{a_2b_2}$, where $\rho_{a_2b_2} = |00\rangle\langle 00|_{a_2b_2}$ is the initial state of two extra ancillary qubits. The additional networks are same as those in part (b) except that the controlled-$\sigma_y$ gate takes the place of controlled-$V_{2k}$ gate. After the additional networks, the output state is

$$\rho_{\rm out}(k) = U_{h_2}U_{\sigma_y}U_{h_2}\big(\rho_{\rm in}^{c}(k)\big)U_{h_2}^{\dagger}U_{\sigma_y}^{\dagger}U_{h_2}^{\dagger}, \tag{16}$$

where $U_{h_2} = H_{a_2} \otimes H_{b_2} \otimes I_{a_1b_1}$ and $U_{\sigma_y} = U_{C-\sigma_y} \otimes U_{C-\sigma_y}$. In equation (16), what we care about is the quantum state evolution of the ancillary qubits $a_2$ and $b_2$. After some deduction, we can get

$$\rho_{a_2b_2}(k) = {\rm Tr}_{a_1b_1}[\rho_{\rm out}(k)] = \frac{1}{4}\begin{pmatrix} 1-\mu_4^{(k)} & 0 & 0 & 0 \\ 0 & 1+\mu_4^{(k)} & 0 & 0 \\ 0 & 0 & 1+\mu_4^{(k)} & 0 \\ 0 & 0 & 0 & 1-\mu_4^{(k)} \end{pmatrix}. \tag{17}$$

If Alice and Bob measure the expectation value of $\sigma_z \otimes \sigma_z$ on the quantum state $\rho_{a_2b_2}(k)$, they will get $\mu_4^{(k)} = -{\rm Tr}[(\sigma_z \otimes \sigma_z)\rho_{a_2b_2}(k)]$.

When the two observers choose different input state $\rho_{\rm in}(k)$ for $k = 2, 3, 4$, they will get the functions $\mu_4^{(2)}, \mu_4^{(3)}, \mu_4^{(4)}$. Inserting equation (10) into equation (13), we can obtain

$$\mu_4^{(k)} = \frac{\gamma}{2}{\rm Tr}[(\rho_{\rm AB}\widetilde{\rho_{\rm AB}})^k] - \frac{\gamma}{4}\Big({\rm Tr}\Big[\big(V_{{\rm A}_{2k}}^{\dagger} \otimes V_{{\rm B}_{2k}}\big)(\rho_{\rm AB} \otimes \widetilde{\rho_{\rm AB}})^{\otimes k}\Big] + {\rm c.c.}\Big), \quad k = 2, 3, 4. \tag{18}$$

Combining equations (14) and (18), we can get

$${\rm Tr}[(\rho_{\rm AB}\widetilde{\rho_{\rm AB}})^k] = \frac{\mu_3^{(k)} + \mu_4^{(k)} - 4\delta}{\gamma}, \qquad k = 2, 3, 4. \tag{19}$$

Once Alice and Bob get all of the seven functions $\mu_3^{(1)}, \mu_3^{(2)}, \mu_3^{(3)}, \mu_3^{(4)}, \mu_4^{(2)}, \mu_4^{(3)}, \mu_4^{(4)}$, they can deduce the parameters ${\rm Tr}[(\rho_{\rm AB}\widetilde{\rho_{\rm AB}})^k]$ for $k = 1, 2, 3, 4$. In terms of the four parameters, Alice and Bob can obtain the eigenvalues $\{\lambda_i\}$, and then the entanglement of formation $E_{\rm f}$. This concludes the description of our LOCC method.

The above method is the LOCC extension of the protocol [10] presented by Horodecki, which is parametrically efficient compared with the LOCC quantum tomography. An unknown two-qubit quantum state $\rho_{\rm AB}$ can be tomographied by LOCC measuring 15 parameters. For example, the quantum state can be expanded as

$$\rho_{\rm AB} = \sum_{ij} \frac{{\rm Tr}\big[\big(\sigma_{{\rm A}_i} \otimes \sigma_{{\rm B}_j}\big)\rho_{AB}\big]\sigma_{{\rm A}_i} \otimes \sigma_{{\rm B}_j}}{4}, \tag{20}$$

where $\sigma_i, \sigma_j = I_2, \sigma_x, \sigma_y, \sigma_z$. In order to reconstruct the quantum state $\rho_{\rm AB}$, Alice and Bob need to measure 15 expectation values ${\rm Tr}\big[\big(\sigma_{{\rm A}_i} \otimes \sigma_{{\rm B}_j}\big)\rho_{\rm AB}\big]$. (Due to the unitary trace property of $\rho_{\rm AB}$, they can subtract one parameter.) However, if they use the LOCC method presented by us, they only need to measure seven parameters.

## 3. Conclusion

In conclusion, we have presented a parametrically efficient LOCC method for measuring two-qubit entanglement of formation without the prior state reconstruction. The work is based on a series of works done by Horodecki, Ekert, Alves and co-workers. In our LOCC method, Alice and Bob first perform three kinds of physical operations on their composite systems (two

SPA of partial transposition operations and one spin-flip operation), by which the requisite quantum states can be prepared. Then they estimate seven functions via two local modified quantum networks supplemented a classical communication. After obtaining all of the seven functions, they can deduce the entanglement of formation $E_{\mathrm{f}}$.

For multilevel bipartite systems, there is no analytical formula for the entanglement of formation. A weaker analytical measure—computable entanglement measure $E_{\mathrm{c}}$ [14] can characterize the distillable entanglement [15]. An efficient method [9] for measuring $E_{\mathrm{c}}$ has been presented as a byproduct of directly checking positive partial transposition (PPT) criterion [16, 17].

There are some entangled states which have the property of PPT. These states represent the bound entanglement [18, 19], which cannot be distillable. Efficient methods for detecting bound entanglement in an unknown quantum state are worthy of consideration. Recently, Doherty *et al* presented the concept of PPT symmetrical extension [20, 21], which makes the solution of the problem possible.

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